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Multiplicity and regularity results for some Lane-Emden systems in unbounded domains

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The aim of this communication is to present some recent results obtained in [1, 2, 3] on the existence, multiplicity and regularity of solutions for some nonlinear elliptic systems of Lane-Emden type in unbounded domains. In [2], we deal with

$$\begin{cases} -\Delta u = sgn(v)|v|^{p-1} & \text{in } \mathbb{R}^N, \\ -\Delta v = -\rho(x) sgn(u)|u|^{\frac{1}{p-1}} + f(x,u) & \text{in } \mathbb{R}^N, \\ u, v \to 0 & \text{as } |x| \to \infty, \end{cases} \tag{0.1}$$

where $N \geq 3$, $1 < p \leq 2$, $\rho : \mathbb{R}^N \to \mathbb{R}$ is a Lebesgue measurable function, positive and satisfying a further asymptotic hypothesis and $f : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ is a continuous function satisfying suitable growth's assumptions at the origin and at infinity. The search of weak solutions for system (0.1) can be reduced to the study of critical points of the energy functional I associated to the equivalent fourth order quasilinear elliptic problem

$$\begin{cases} -\Delta(-\Delta u)^{\frac{1}{p-1}} + \rho(x) u^{\frac{1}{p-1}} = f(x,u) & \text{in } \mathbb{R}^N, \\ u, \Delta u \to 0 \quad \text{per } |x| \to +\infty. \end{cases} \tag{0.2}$$

In order to overcome the lack of compactness of the problem, by means of a result by [4], we prove some compact imbeddings of suitable weighted Sobolev spaces. A direct application of Mountain Pass Theorem allows us to establish the existence of one nontrivial weak solution of the system. Under further hypotheses of symmetry for f, we prove a multiplicity result by the Symmetric Version of Mountain Pass Theorem. Moreover, in [3], by modifying some arguments in [7] we prove the regularity of the weak solutions of system (0.1) under stronger regularity assumptions for ρ and f.
Always in [3], we study (0.1) when $f(x,u) = u^{q-1} + h(x)$ with $h : \mathbb{R}^N \to \mathbb{R}$, namely

$$\begin{cases} -\Delta u = sgn(v)|v|^{p-1} & \text{in } \mathbb{R}^N, \\ -\Delta v = -\rho(x) sgn(u)|u|^{\frac{1}{p-1}} + u^{q-1} + h(x) & \text{in } \mathbb{R}^N, \\ u, v \to 0 & \text{as } |x| \to \infty, \end{cases} \tag{0.3}$$

with p, q verifying suitable conditions. When the perturbation term h is small enough, we look for critical points of the energy functional associated to the system by following Pohozaev Fibering Method [9] and we prove the existence of at least three solutions of (0.3) that are classical if ρ and h are smooth.
More recently, in [1] we deal with

$$\begin{cases} -\Delta u = sgn(v)|v|^{p-1} & \text{in } \Omega, \\ -\Delta v = -\lambda sgn(u)|u|^{\frac{1}{p-1}} + f(x,u) & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases} \tag{0.4}$$

where $\lambda \in \mathbb{R}$ and Ω is an unbounded cylinder i.e., $\Omega = \widetilde{\Omega} \times \mathbb{R}^{N-m} \subset \mathbb{R}^N$, $N-m \geq 2$, $\widetilde{\Omega} \subset \mathbb{R}^m$ ($m \geq 1$) open bounded. Here, by means of a result by P.L. Lions [8], we overcome the lack of compactness of the problem by proving some compact imbeddings of suitable "partially" spherically symmetric Sobolev spaces. So, under spherically symmetric assumptions on f in $\mathbb{R}^{N-m}$, by the Principle of Symmetric Criticality by Palais, we can look for critical points

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of I constrained on such spaces. In particular, a finite-dimensional decomposition of these spaces is obtained for suitable $p's$ by following an approach introduced in [5]. Mountain Pass Theorem and its Symmetric versions allow us to establish existence and multiplicity results when $\lambda \in \mathbb{R}$. Regularity of solutions of (0.4) follows as in [3] if f is smooth enough.

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