WNDE - Workshop in Nonlinear Differential Equation
UFPB - Universidade Federal da Paraíba
Edition IX, João Pessoa - Brazil, September 17-21, 2012

Nonexistence and multiplicity of solutions to critical and supercritical elliptic problems

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We consider the problem

$$-\Delta u = |u|^{p-2}\, u \ \text{ in } \ \Omega, \quad u = 0 \ \text{ on } \ \partial\Omega,$$

where Ω is a bounded smooth domain in $\mathbb{R}^N$, $N \geq 3$, and $p \geq 2^* := \frac{2N}{N-2}$.

For $p = 2^*$ Bahri and Coron showed that, if the homology of Ω is nontrivial, the problem has a positive solution. However, this condition is not enough to guarantee existence in the supercritical case: for $p \geq \frac{2(N-1)}{N-3}$, $N \geq 4$, Passaseo exhibited domains carrying one nontrivial homology class, in which no nontrivial solution exists.

We shall give examples of domains whose homology becomes richer as p increases, in which no nontrivial solution exists. We shall also present some new multiplicity results for this problem, both in the critical and the supercritical case.

This is joint work with Jorge Faya (Universidad Nacional Autónoma de México) and Angela Pistoia (Università di Roma "La Sapienza").

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