

# POSITIVE SOLUTIONS FOR THE $p$ -LAPLACIAN INVOLVING CRITICAL AND SUPERCRITICAL NONLINEARITIES WITH ZEROS

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In this talk we show the existence of multiple solutions to a class of quasilinear elliptic equations when the continuous nonlinearity has a positive zero and it satisfies a  $p$ -linear condition only at zero. In particular, our approach allows us to consider superlinear, critical and supercritical nonlinearities.

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