

A BACKWARD LAMBDA LEMMA FOR THE FORWARD HEATFLOW

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Fix a closed Riemannian manifold M and consider the energy functional and the corresponding heatflow on the free loop space of M . After suitable perturbation the energy functional is Morse and we obtain an infinite dimensional hyperbolic dynamical system whose fixed points are the (perturbed) closed geodesics x in M . The stable manifold of x is embedded locally near x , though of infinite dimension in general. The unstable manifold of x , suitably defined, is globally embedded and its dimension is finite and given by the Morse index of x . All this is well known.

Now choose a disk transversal to the unstable manifold and recall the lambda lemma known in finite dimensions. It asserts that under the backward flow the chosen disk converges uniformly in C^1 to the stable manifold of x . The obvious first problem encountered in the infinite dimensional situation at hand is the absence of a backward flow. In this talk we, firstly, propose a suitable replacement and, secondly, establish a backward lambda lemma for the heatflow. Convergence being exponential and uniform on an (infinite dimensional) disk.

This backward lambda lemma is a major ingredient in our construction [1] of a natural isomorphism between Morse homology associated to the heatflow and singular homology of the free loop space. In fact, this is how we came across the needs for a backward lambda lemma.

References

- [1] J. WEBER, *Conley index theory and the heat flow*. In preparation.

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