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Multiple radial solutions for Neumann problems involving the mean extrinsic curvature operator in Minkowski space

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We use critical point theory for convex, lower semicontinuous perturbations of C^1-functionals to establish existence of multiple radial solutions in a ball or an annulus $\mathcal{A} = \{x \in \mathbb{R}^N : R_1 \leq |x| \leq R_2\}$ $(0 \leq R_1 < R_2)$ for some Neumann problems involving the mean extrinsic curvature operator in Minkowski space $\mathcal{M}(v) := \mathrm{div}\left(\frac{\nabla v}{\sqrt{1-|\nabla v|^2}}\right)$.

In what follows $\alpha > 0$, $p > q \geq 2$, $m \geq 2$, $\lambda > 0$, $b, h : [R_1, R_2] \to \mathbb{R}$, $f : [R_1, R_2] \times \mathbb{R} \to \mathbb{R}$ are continuous, and $F(r,u) := \int_0^u f(r,v)\,dv$ is such that

$$\limsup_{|x|\to 0} \frac{pF(r,x)}{|x|^p} < \alpha \quad uniformly\ in\ r \in [R_1, R_2]. \tag{0.1}$$

and that *for some $\theta > p$ and $x_0 > 0$ one has*

$$0 < \theta F(r,x) \leq x f(r,x) \quad for\ all \quad r \in [R_1, R_2] \quad and \quad |x| \geq x_0. \tag{0.2}$$

Teorema 0.1. *If, in addition to (0.1) and (0.2),*

(i) either $\liminf_{x\to 0-} \frac{F(r,x)}{|x|^p} \geq 0$ *uniformly in* $r \in [R_1, R_2]$
or $\liminf_{x\to 0+} \frac{F(r,x)}{x^p} \geq 0$ *uniformly in* $r \in [R_1, R_2]$

(ii) $\int_{R_1}^{R_2} r^{N-1} b(r)\,dr > 0,$

problem

$$-\mathcal{M}(v) + \alpha |v|^{p-2} v = f(|x|, v) + \lambda b(|x|)|v|^{q-2} v \quad in\ \mathcal{A}, \quad \frac{\partial v}{\partial \nu} = 0 \quad on \quad \partial\mathcal{A},$$

has at least two nontrivial solutions for sufficiently small values of λ.

Those assumptions correspond to problems with *convex-concave nonlinearities*.

Teorema 0.2. *If, in addition to (0.1) and (0.2),*

(i)′ there exists $k_1, k_2 > 0$ and $0 < \sigma < m$ such that
$-l(r) \leq F(r,x) \leq k_1|x|^\sigma + k_2,$ *for all* $(r,x) \in [R_1, R_2] \times \mathbb{R},$
where $l \geq 0$ is measurable and $\displaystyle\int_{R_1}^{R_2} r^{N-1} l(r)dr < +\infty;$

(ii)′ either $\lim_{|x|\to\infty} \int_{R_1}^{R_2} r^{N-1} F(r,x)dr = +\infty,$
or $F_\pm(r) := \lim_{x\to\pm\infty} F(r,x)$ *exist for all* $r \in [R_1, R_2]$ *and*
$F(r,x) < F_+(r), \quad \forall r \in [R_1, R_2],\ x \geq 0,$
$F(r,x) < F_-(r), \quad \forall r \in [R_1, R_2],\ x \leq 0$

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(iii)' $\int_{R_1}^{R_2} r^{N-1}h(r)dr = 0,$

problem

$$-\mathcal{M}(v) + \lambda|v|^{m-2}v = f(|x|, v) + h(|x|) \quad \text{in } \mathcal{A}, \quad \frac{\partial v}{\partial \nu} = 0 \quad \text{on} \quad \partial\mathcal{A}$$

has at least three nontrivial solutions for sufficiently small values of λ.

Results of this type are called *multiplicity results near resonance.*

This is a joint work with Cristian Bereanu and Petru Jebelean.