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ON A RESONANT PROBLEM FOR THE p-LAPLACIAN *

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In this talk we address the following resonant problem

$$\begin{cases} -\Delta_p u = \lambda_p |u|^{q-2} u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{0.1}$$

in a bounded and smooth domain Ω, where λ_p is the first eigenvalue of the p-Laplacian operator Δ_p, $p > 1$, and $q \in [1, p) \cup (p, p^\star)$.

The main result is the following:

Teorema 0.1. *Let $\{u_q\}_{q\in[1,p)\cup(p,p^\star)}$ be an arbitrary family of positive solutions of (0.1). The convergence $u_q \to \theta_p e_p$ holds in $C^1(\overline{\Omega})$ as $q \to p$, where*

$$\theta_p := \exp\left(\|e_p\|_p^{-p} \int_\Omega e_p^p |\ln e_p| \, dx \right)$$

and e_p is the positive and L^∞-normalized first eigenfunction of the p-Laplacian.

As a consequence of this result the function $q \in [1, p^\star) \to \lambda_q := \min \left\{ \dfrac{\|\nabla u\|_p^p}{\|u\|_q^p} : u \in W_0^{1,p}(\Omega)/\{0\} \right\}$ is differentiable at $q = p$ and

$$\frac{d}{dq} [\lambda_q]_{q=p} = \lambda_p \ln(\theta_p \|e_p\|_p).$$

Moreover, the asymptotic behavior (as $q \to p$) of positive solutions $u_{\lambda,q}$ of the Lane-Emden problem

$$\begin{cases} -\Delta_p u = \lambda |u|^{q-2} u & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

is completely determined in the space $C^1(\overline{\Omega})$, for each $\lambda > 0$.

As the same time, we also prove that the function $q \in [1, p^\star) \to \lambda_q |\Omega|^{\frac{p}{q}}$ is strictly decreasing and Lipschitz continuous in each closed interval contained in $[1, p^*)$. Moreover, if $1 < p < N$ this function is absolutely continuous in the closed interval $[1, p^\star]$.

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