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Multiple Solutions for Nonvariational Singular Elliptic Equations. A Monotonicity Approach.

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We consider problems of the type:

$$\sum_{ij}(a_{ij}(x)D_iD_ju + b_i(x)D_iu) + g(x,u) = f(x,u) \text{ in } \Omega, \qquad u = 0 \text{ on } \partial\Omega$$

where the linear part is uniformly elliptic (not necessarily symmetric) and where $g(x,u)$ is increasing and singular in u. We regard the problem as $\mathcal{A}_g(u) = i \circ \mathcal{N}_f(u)$ where $\mathcal{A}_g$ is a maximal monotone operator ($\mathcal{A}_g u = -\text{div}(a\nabla u) + b\nabla u + \partial\mathcal{G}(u)$, $G'_s(x,s) = g(x,s)$), i is a suitable (compact) embedding, and $\mathcal{N}_f$ is the Nemetskii operator relative to f. This allows to look for (weak) solutions as fixed points of $id - \mathcal{R}_g \circ i \circ \mathcal{N}_f$, $\mathcal{R}_g$ being the resolvent of $\mathcal{A}_g$ (provided the elliptic operator is coercive). This setting is also quite effective if sub or super solutions of the problem are known. In this case we can use them as "natural constraints" including them, in some sense, in the operator $\mathcal{A}_g$, and look for solutions of the original problem lying above a subsolution or below a supersolution. The latter is also useful for providing regularity of the solutions.

We use this approach to prove that, if $q < 0$, $1 < p < 2^* - 1$, under suitable regularity assumptions on a_{ij}, b_i, the problem:

$$\sum_{ij}(a_{ij}(x)D_iD_ju + b_i(x)D_iu) - u^q = \lambda u^p \text{ in } \Omega, \qquad u = 0 \text{ on } \partial\Omega$$

has two solutions in $\mathcal{C}^2(\Omega) \cap \mathcal{C}^0(\bar{\Omega})$, for λ small. To get this result we need to prove a uniform bound of the solutions of the problem, when λ stays away from zero.

With the same ideas we can also prove a three solutions result for problems with sublinear (singular) nonlinearities.

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