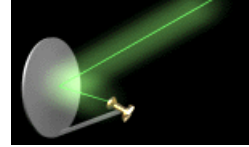


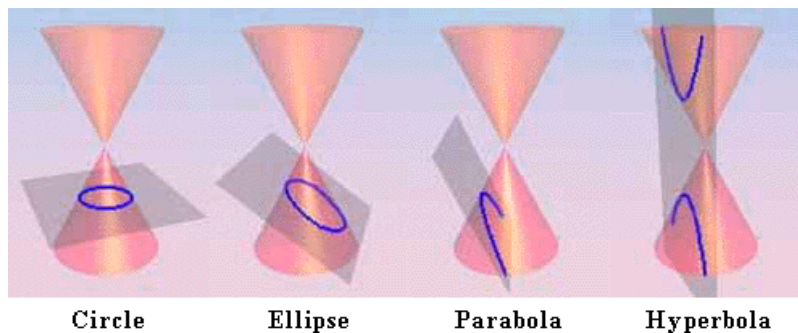
# OCCURRENCE OF THE CONICS



Mathematicians have a habit of studying, just for the fun of it, things that seem utterly useless; then centuries later their studies turn out to have enormous scientific value.

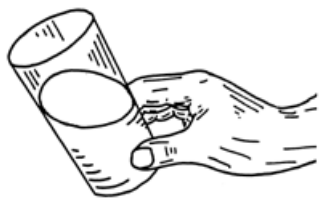
There is no better example of this than the work done by the ancient Greeks on the curves known as the conics: the ellipse, the parabola, and the hyperbola. They were first studied by one of Plato's pupils. No important scientific applications were found for them until the 17th century, when Kepler discovered that planets move in ellipses and Galileo proved that projectiles travel in parabolas.

Appolonios of Perga, a 3rd century B.C. Greek geometer, wrote the greatest treatise on the curves. His work "Conics" was the first to show how all three curves, along with the circle, could be obtained by slicing the same right circular cone at continuously varying angles.



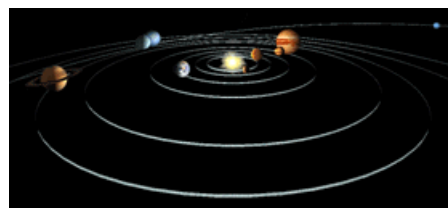
## THE ELLIPSE

Though not so simple as the circle, the ellipse is nevertheless the curve most often "seen" in everyday life. The reason is that every circle, viewed obliquely, appears elliptical.

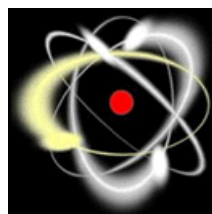
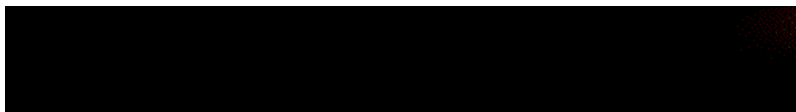


Tilt a glass of water and the surface of the liquid acquires an elliptical outline. Salami is often cut obliquely to obtain elliptical slices which are larger.

The early Greek astronomers thought that the planets moved in circular orbits about an unmoving earth, since the circle is the simplest mathematical curve. In the 17th century, Johannes Kepler eventually discovered that each planet travels around the sun in an elliptical orbit with the sun at one of its foci.

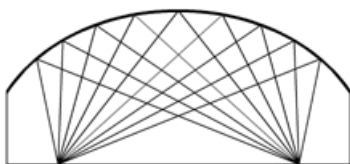
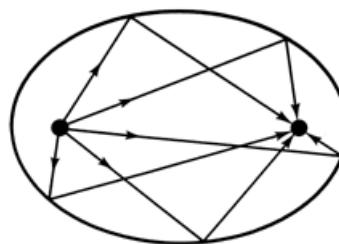


The orbits of the moon and of artificial satellites of the earth are also elliptical as are the paths of comets in permanent orbit around the sun. *Halley's Comet* takes about 76 years to travel around our sun. Edmund Halley saw the comet in 1682 and correctly predicted its return in 1759. Although he did not live long enough to see his prediction come true, the comet is named in his honour.



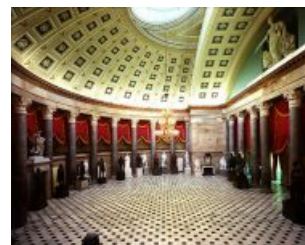
On a far smaller scale, the electrons of an atom move in an approximately elliptical orbit with the nucleus at one focus.

The ellipse has an important property that is used in the reflection of light and sound waves. Any light or signal that starts at one focus will be reflected to the other focus. This principle is used in *lithotripsy*, a medical procedure for treating kidney stones. The patient is placed in an elliptical tank of water, with the kidney stone at one focus. High-energy shock waves generated at the other focus are concentrated on the stone, pulverizing it.



The principle is also used in the construction of "whispering galleries" such as in St. Paul's Cathedral in London. If a person whispers near one focus, he can be heard at the other focus, although he cannot be heard at many places in between.

Statuary Hall in the U.S. Capital building is elliptical. It was in this room that John Quincy Adams, while a member of the House of Representatives, discovered this acoustical phenomenon. He situated his desk at a focal point of the elliptical ceiling, easily eavesdropping on the private conversations of other House members located near the other focal point.

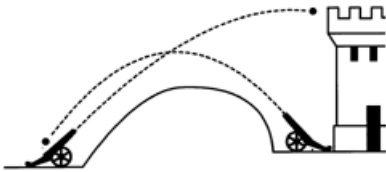




The ability of the ellipse to rebound an object starting from one focus to the other focus can be demonstrated with an elliptical pool table. When a ball is placed at one focus and is thrust with a cue stick, it will rebound to the other focus. If the pool table is live enough, the ball will continue passing through each focus and rebound to the other.

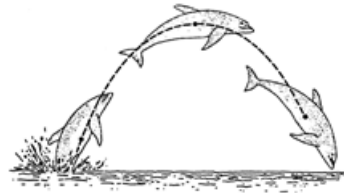
## THE PARABOLA

One of nature's best known approximations to parabolas is the path taken by a body projected upward and obliquely to the pull of gravity, as in the parabolic trajectory of a golf ball. The friction of air and the pull of gravity will change slightly the projectile's path from that of a true parabola, but in many cases the error is insignificant.



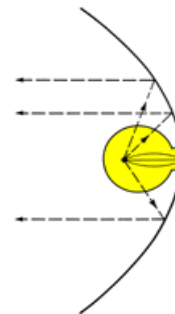
This discovery by Galileo in the 17th century made it possible for cannoners to work out the kind of path a cannonball would travel if it were hurtled through the air at a specific angle.

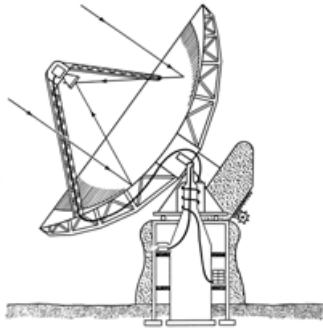
When a baseball is hit into the air, it follows a parabolic path; the center of gravity of a leaping porpoise describes a parabola.



The easiest way to visualize the path of a projectile is to observe a waterspout. Each molecule of water follows the same path and, therefore, reveals a picture of the curve.

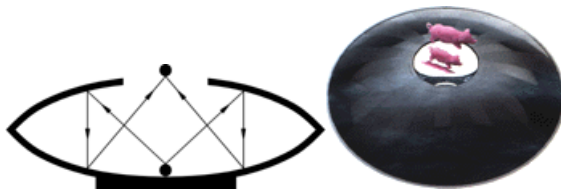
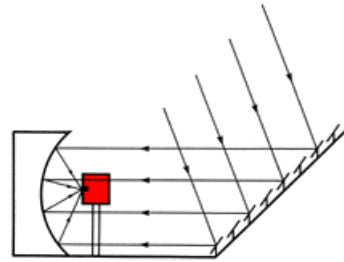
Parabolas exhibit unusual and useful reflective properties. If a light is placed at the focus of a parabolic mirror (a curved surface formed by rotating a parabola about its axis), the light will be reflected in rays parallel to said axis. In this way a straight beam of light is formed. It is for this reason that parabolic surfaces are used for headlamp reflectors. The bulb is placed at the focus for the high beam and in front of the focus for the low beam.





The opposite principle is used in the giant mirrors in reflecting telescopes and in antennas used to collect light and radio waves from outer space: the beam comes toward the parabolic surface and is brought into focus at the focal point. The largest parabolic mirror in existence is in a telescope located in the Caucasus mountains in Russia. It is nearly 20 feet in diameter and was completed in 1967.

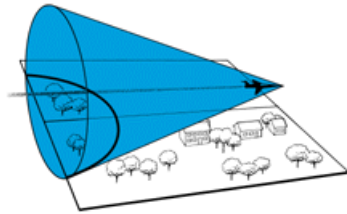
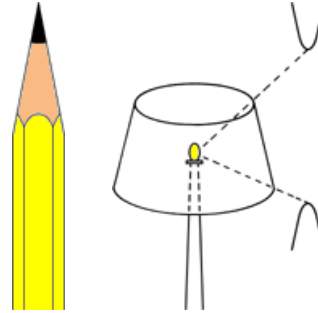
Heat waves, as well as light and sound waves, are reflected to the focal point of a parabolic surface. If a parabolic reflector is turned toward the sun, flammable material placed at the focus may ignite. (The word "focus" comes from the Latin and means *fireplace*.) A solar furnace produces heat by focusing sunlight by means of a parabolic mirror arrangement. Light is sent to it by set of moveable mirrors computerized to follow the sun during the day.



Two types of images exist in nature: real and virtual. In a real image, the light rays actually come from the image. In a virtual image, they appear to come from the reflected image - but do not. For example, the virtual image of an object in a flat mirror is "inside" the mirror, but light rays do not emanate from there. Real images can form "outside" the system, where emerging light rays cross and are caught - as in a *Mirage*, an arrangement of two concave parabolic mirrors. *Mirage's* 3-D illusions are similar to the holograms formed by lasers.

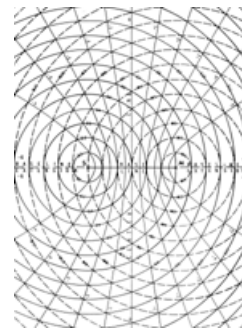
## THE HYPERBOLA

If a right circular cone is intersected by a plane perpendicular to its axis, part of a hyperbola is formed. Such an intersection can occur in physical situations as simple as sharpening a pencil that has a polygonal cross section or in the patterns formed on a wall by a lamp shade.



A sonic boom shock wave has the shape of a cone, and it intersects the ground in part of a hyperbola. It hits every point on this curve at the same time, so that people in different places along the curve on the ground hear it at the same time. Because the airplane is moving forward, the hyperbolic curve moves forward and eventually the boom can be heard by everyone in its path.

All three conic sections can be characterized by moiré patterns. If the center of each of two sets of concentric circles is the source of a radio signal, the synchronized signals would intersect one another in associated hyperbolas. This principle forms the basis of a hyperbolic radio navigation system known as *Loran* (**L**ong **R**ange **N**avigation).



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