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Title: Boundary Controlability of a Unidimensional Phase-Field System with one Control Force

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Abstract: We deal with the controllability properties of the following phase-field system which is a model describing the transition between the solid and liquid phase in solidification (liquidification) process to a matter occupying a region.

$$\begin{cases} \theta_t - \xi\theta_{xx} + \dfrac{1}{2}\rho\xi\phi_{xx} + \dfrac{\rho}{4\tau}(\phi - \phi^3) + \dfrac{\rho}{\tau}\theta = 0 & \text{in} \quad Q_T, \\ \phi_t - \xi\phi_{xx} - \dfrac{1}{2\tau}(\phi - \phi^3) - \dfrac{2}{\tau}\theta = 0 & \text{in} \quad Q_T, \\ \theta = v\mathbf{1}_{\Gamma_0}\ , \phi = c & \text{on} \quad \Sigma_T, \\ \theta(0) = \theta_0\ , \phi(0) = \phi_0 & \text{in} \quad (0,\pi), \end{cases} \tag{1}$$

This system models the temperature and the solidification level of the material and suggests a controllability with only one control force acting on the temperature variable. With a spectral study of the stationary operator of the linearized system we reduce the controllability issue to a use the moment problem. Given some conditions concerning the constants ξ, ρ, τ, each one with its physical meanings, we solve this moment problem and proceed to deal with the non linear system. Finally, we show that the system above is null controllable in the sense that the we can reach a full solid state or full liquid state.