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Averaging principle on foliated space: application to the topology of submanifolds

Abstract

Consider an SDE on a foliated manifold such that each trajectory lays on a single leaf of the foliation. We investigate the effective behaviour of a perturbation of order ϵ in a direction transversal to the leaves, hence destroying the foliated structure of the trajectories. An average principle is shown to hold such that, rescaling the time, the vertical (transversal) coordinate converges to the solution of a deterministic ODE, according to the average of the perturbing vector field with respect to invariant measures on the leaves, as ϵ goes to zero. An estimate of the rate of convergence is given. These results generalize the geometrical scope of previous approaches, including completely integrable stochastic Hamiltonian system (X.M. Li, Nonlinearity 2008). We apply this result to Brownian motion on compact submanifolds of Euclidean spaces, such that the coefficients of the corresponding deterministic ODE's in the vertical coordinate are given by the Euler characteristics of the submanifolds.