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CONTROLLABILITY FOR A COUPLED PHASE FIELD SYSTEM FOR SOLIDIFICATION BY ONE CONTROL FUNCTION

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In this work it is investigated a null controllability problem for a phase field system modeling the solidification process of pure materials in the case that only one control force is used. The system is constituted of one energy balance equation, with a localized control associated to the density of heat sources and sinks to be determined, coupled with a phase field equation with the classical nonlinearity derived from the two-well potential. We prove that this system has the local null controllability property.

The mentioned system is given by

$$\begin{cases} u_t - \Delta u + l\phi_t = v\mathbf{1}_{\mathcal{O}} & \text{in} \quad Q, \\ \phi_t - \Delta\phi - \left(a\phi + b\phi^2 - \phi^3\right)\phi = u & \text{in} \quad Q, \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial \phi}{\partial \nu} = 0 & \text{on} \quad \partial\Omega, \\ u\,(0) = u_0, \quad \phi\,(0) = \phi_0 & \text{in} \quad \Omega, \end{cases} \tag{0.1}$$

where $\Omega \subset \mathbb{R}^3$ is a bounded open set with a regular boundary, $\mathcal{O} \subset Q$ be a (small) nonempty open subset. For $T > 0$, it is considered the cylindrical domain $Q = \Omega \times (0, T)$. By $\nu = \nu\,(x)$ we denote the outward unit normal vector to Ω at a point $x \in \partial\Omega$.

Here, function $u = u\,(x, t)$ is related to the temperature of the material, $\phi = \phi(x, t)$ is the phase field functions used to identify the level of solid crystallization in (x, t) and v is a control function to be determined, which corresponds to the density of heat sources and sinks to be applied in $\mathcal{O}$ and it is called control function. Besides, $\mathbf{1}_{\mathcal{O}}$ denotes the characteristic function of $\mathcal{O}$, l, a and b are constants depending on the physical properties of the material and the initial data u_0 and ϕ_0 are suitable given functions.

Existence, uniqueness and regularity of solution for phase field system (0.1) was proved by Hoffman and Jiang in [1]. They also studied a optimal control problem for (0.1).

1 Main Result

Teorema 1.1. *There exists $r_0 > 0$ such that for any data $(u_0, \phi_0) \in [W_s^{2-2/s}(\Omega) \cap \{w \in H^2\,(\Omega) : \ \partial w/\partial\nu = 0 \text{ on } \partial\Omega\}]^2$, with $s > 5/2$, satisfying*

$$\|(u_0, \phi_0)\|_{[W_s^{2-2/s}(\Omega)]^2} < r_0,$$

there exists a control function $v \in L^2\,(\Omega)$ such that the solution (u, ϕ) of (0.1) satisfies

$$u\,(\cdot, T) = \phi\,(\cdot, T) = 0 \quad \textit{in} \quad \Omega.$$

Reference

[1] K. HOFFMAN AND L. JIANG, Optimal Control of a Phase Field Model for Solidification, *Numer. Funct. Anal. and Optimiz.*, **13**, 11-27, 1992.

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