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# About some linear non local problems

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Let $\Omega$ be a bounded set in $\mathbb{R}^{\mathbb{N}}$,

$$\begin{cases} u_t(x,t) = L(u)(x,t) = K(u)(x,t) - h(x)u(x,t), & x \in \Omega,\ t > 0 \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{0.1}$$

with $K(u)(x,t) = \displaystyle\int_\Omega J(x,y)u(y,t)dy$, $h$ a bounded function and $u_0 \in L^p(\Omega)$, for $1 \leq p \leq \infty$. In particular for the Neumann problem $h(x) = h_0(x) = \displaystyle\int_\Omega J(x,y)dy$, see [1, 2].

We study the asymptotic behaviour of the solution of (0.1). We have that $L \in \mathcal{L}(L^p(\Omega), L^p(\Omega))$ and assume that the spectrum $\sigma(L)$ is a disjoint union of two closed subsets $\sigma_1$ and $\sigma_2$, with $\delta_2 < \mathrm{Re}(\sigma_1) \leq \delta_1$, $\mathrm{Re}(\sigma_2) \leq \delta_2$, for $\delta_2 < \delta_1$.
Consider that $Q_{\sigma_1}$ is a spectral projection over the space generated by the eigenfunctions associated to $\sigma_1$.

**Theorem 0.1.** *Considering all the hypotheses we have made before. The solution of (0.1) satisfies that*

$$\lim_{t\to\infty} \|e^{-\mu t}\left(u(x,t) - Q_{\sigma_1}(u)(x,t)\right)\|_{L^p(\Omega)} = 0,\ \forall \mu > \delta_2.$$

Let us see in particular, the asymptotic behaviour of the solution of (0.1), with $h$ constant and $h = h_0$, considering $\sigma_1 = \{\lambda_1\}$, with $\lambda_1$ the first eigenvalue of the operator $L$ isolated and simple, whose associated eigenfunction is $\Phi_1$.

**Corollary 0.2.** *For $h = a$ constant and $p = \infty$, if the initial data $u_0$ is bounded then we have the following asymptotic behavior*

$$\lim_{t\to\infty} \max_{x\in\Omega} |e^{-\lambda_1 t}u(x,t) - C^*\Phi_1(x)| = 0$$

*with* $C^* = \dfrac{\int_\Omega u_0(x)\Phi_1(x)dx}{\int_\Omega \Phi_1^2(x)dx}$.

**Corollary 0.3.** *For the "Neumann" problem with $h = h_0$ and $p = \infty$, if the initial data $u_0$ is bounded, then we have the following asymptotic behavior*

$$\lim_{t\to\infty} \max_{x\in\Omega} \left| e^{-(\mu_2+\delta)t}\left(u(x,t) - \frac{1}{|\Omega|}\int_\Omega u(x)dx\right)\right| = 0,$$

*with $\mu_2$ the second eigenvalue of the operator $K - h_0$, and $\delta > 0$.*

**Remark 0.4.** *The Corollaries 0.2 and 0.3 allow us recovering in a more general way, the asymptotic results in [2].*

## References

[1] ANDREU F., MAZON J.M., ROSSI J. D., TOLEDO J., The Neumann problem for nonlocal nonlinear diffusion equations, Journal of Evolution Equations. Vol. 8(1), 189-215, (2008).

[2] CHASSEIGNE, E., CHAVES, M., ROSSI, J.D., Asymptotic behavior for nonlocal diffusion equations, J. Math. Pures Appl., 86, 271-291, (2006).

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