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WELL-POSEDNESS FOR A FAMILY OF PERTURBATIONS OF THE KDV EQUATION IN PERIODIC SOBOLEV SPACES

XAVIER CARVAJAL* & RICARDO A. PASTRÁN†

We consider the λ-periodic Cauchy problem for

$$\begin{cases} u_t + u_{xxx} + \eta L u + u u_x = 0, & x \in [0,\lambda], \ \ t \in [0,+\infty), \\ \qquad\qquad u(x,0) = u_0(x), \end{cases} \tag{0.1}$$

where $\eta > 0$ is a constant, the linear operator L is defined via the Fourier transform by

$$(Lu)^\wedge(k) = -\Phi(k)\widehat{u}(k), \qquad \text{where} \quad k \in \mathbb{Z}/\lambda, \tag{0.2}$$

and the Fourier symbol $\Phi(k)$ is a real valued function which is bounded above; i.e., there is a constant α such that $\Phi(k) \leq \alpha$. Particular cases of this problem are the Korteweg-de Vries-Burgers equation [5], for $\Phi(k) = -k^2$, the derivative Korteweg-de Vries-Kuramoto-Sivashinsky equation for $\Phi(k) = k^2 - k^4$, and the Ostrovsky-Stepanyams-Tsimring equation for $\Phi(k) = |k| - |k|^3$. To study this Cauchy problem we will be based on the theory developed by Bourgain [1] and Kenig, Ponce and Vega [4]. The initial value problem (0.1) is a generalized perturbation of the KdV equation proposed by Carvajal and Panthee in [2]. They proved local well-posedness in Sobolev spaces $H^s(\mathbb{R})$, with $s > -3/4$, when the symbol Φ is given by

$$\Phi(\xi) = \sum_{j=0}^{n} \sum_{l=0}^{2m} C_{l,j}\, \xi^l \, |\xi|^j; \quad C_{l,j} \in \mathbb{R}, \ \ C_{2m,n} = -1.$$

Note that the index $s = -3/4$ is the critical index for the real KdV equation. They used the usual Bourgain's space associated to the KdV equation instead of the Bourgain's space associated to the linear part of the initial value problem (0.1). We will show that is possible, following the idea of Carvajal and Panthee, to prove that (0.1) is locally well posed in periodic Sobolev spaces $H^s(\mathbb{T})$, with $s > -1/2$. Note that the index $s = -1/2$ is the critical index for the periodic KdV equation. However, we will prove, via the results from Colliander, Keel, Staffilani, Takaoka and Tao in [3] and [6], that

Theorem 0.1. *The initial value problem (0.1) with $\eta > 0$ and L given by (0.2) is locally well-posed for any data $u_0 \in H^s(\mathbb{T})$, for $s \geq -1/2$.*

References

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*Instituto de Matemática , UFRJ, RJ, Brasil, e-mail: carvajal@im.ufrj.br
†Instituto de Matemática , UFRJ, RJ, Brasil, e-mail: rapastranr@unal.edu.co