EBED - Escola Brasileira de Equações Diferenciais
UFPB - Universidade Federal do Paraíba
Edição N$^{\underline{o}}$ 4 Agosto 2011

NONHOMOGENEOUS BOUNDARY CONDITIONS AS LIMIT OF TERMS CONCENTRATING IN THE BOUNDARY

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Let $\Omega = (0,1) \times (0,1)$, $\Gamma = \{(x_1, 0) \in \mathbb{R}^2 : 0 \leqslant x_1 \leqslant 1\}$ and $g \in L^\infty(\mathbb{R})$, T-periodic, with $0 < g_0 \leqslant g(x) \leqslant g_1$ almost every $x \in \mathbb{R}$. The mean value of g over $(0,T)$ is the real number given by $M(g) = \frac{1}{T}\int_0^T g(s)ds$. We define

$$\omega_\epsilon = \left\{(x_1, x_2) \in \mathbb{R}^2 : 0 \leqslant x_1 \leqslant 1 \text{ and } 0 \leqslant x_2 < \epsilon g\left(\tfrac{x_1}{\epsilon}\right)\right\},$$

for sufficiently small ϵ, say $0 < \epsilon \leqslant \epsilon_0$. We denote by $\mathcal{X}_{\omega_\epsilon}$ the characteristic function of the set ω_ϵ. We note that for small ϵ, the set ω_ϵ is a neighborhood of Γ in $\bar{\Omega}$, that collapses to the boundary when the parameter ϵ goes to zero. We observe that the upper boundary of the set ω_ϵ presents a highly oscillatory behaviour and, moreover, the height of the ω_ϵ, the amplitude and period of the oscillations are all of the same order, given by the small parameter ϵ.

We are interested in the behaviour, for small ϵ, of the solutions of the elliptic problem, when some reaction and potential terms are concentrated in ω_ϵ,

$$\begin{cases} -\Delta u^\epsilon + \lambda u^\epsilon + \frac{1}{\epsilon}\mathcal{X}_{\omega_\epsilon} V_\epsilon u^\epsilon = \frac{1}{\epsilon}\mathcal{X}_{\omega_\epsilon} f, & \Omega \\ \frac{\partial u^\epsilon}{\partial n} = 0, & \partial\Omega. \end{cases} \tag{0.1}$$

Assuming that $f \in H^1(\Omega)$, that the family of functions V_ϵ, $0 < \epsilon \leqslant \epsilon_0$, satisfies

$$\frac{1}{\epsilon}\int_{\omega_\epsilon} |V_\epsilon|^2 \leqslant C,$$

for some $C > 0$ independent of ϵ, and that there exists a function $V_0 \in L^2(\Gamma)$ such that

$$\lim_{\epsilon \to 0} \frac{1}{\epsilon}\int_{\omega_\epsilon} V_\epsilon \varphi = \int_\Gamma V_0 \varphi, \qquad \forall\ \varphi \in C^\infty(\bar{\Omega}),$$

we will show that, for sufficiently large λ, the solutions of (**??**) converge in $H^1(\Omega)$ to the unique solution of

$$\begin{cases} -\Delta u^0 + \lambda u^0 = 0, & \Omega \\ \frac{\partial u^0}{\partial n} + V_0 u^0 = M(g) f, & \Gamma \\ \frac{\partial u^0}{\partial n} = 0, & \partial\Omega \setminus \Gamma. \end{cases} \tag{0.2}$$

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