ENERGY ESTIMATES AT INFINITY FOR HYPERBOLIC-LIKE DISSIPATION EQUATIONS.

MARCELO R. EBERT*

The goal of this talk is to discuss about the long time behavior of the energy to the strictly hyperbolic Cauchy problem

\[ \partial^m_t u - \sum_{j=1}^{m} a_j(t)\lambda^j(t)\partial^{m-j}_t \partial^j_x u + \sum_{j+k \leq m-1} c_{j,k}(t)\lambda^k(t)\partial^j_t \partial^k_x u = 0, \quad (t, x) \in (0, \infty) \times \mathbb{R}, \]

\[ \partial^j_t u(0, x) = u_j(x), \quad j = 0, 1, \ldots, m - 1. \]

(0.1)

For sake of simplicity, we consider this model problem in one space dimension, but our arguments can be immediately extended to the case \( x \in \mathbb{R}^n, n \geq 2 \).

It is well known (see, e.g., [1]) that if the coefficients are sufficiently regular and bounded, then the Cauchy problem (0.1) is \( C^\infty \) well-posed. More precisely, in this case we have well-posedness in Sobolev spaces and for any given Cauchy data \( u_j \in H^{s+m-1-j}(\mathbb{R}), s \in \mathbb{R}, \) there is a unique solution \( u \in m-1 \bigcap_0 C^j([0, \infty); H^{s+m-1-j}) \).

The problem of sharp decay estimate goes back to Strichartz-type decay estimate ([2]). Later A. Matsumura ([3]) proved sharp decay estimates for the damped wave equations. They proved the results by using WKB-representation of the solutions. More recently, the influence of a time-dependent coefficient on such decay estimate for the wave equation and dissipative wave equations was studied in a series of papers [4], [5] and [6].

To obtain an energy estimate to the Cauchy problem (0.1), our approach will be, by using the hyperbolicity, to state a very precise energy behavior in some region of the extend phase space, then we state conditions, namely, Hyperbolic-like Dissipation, in order to have the same energy estimate in all the extend phase space.

This is a joint work with Marcello D’Abbicco.

References


*Departamento de Computação e Matemática, Universidade de São Paulo, SP, Brasil, ebert@ffclrp.usp.br