EBED - Escola Brasileira de Equações Diferenciais
UFPB - Universidade Federal do Paraíba
Edição Nº 4 Agosto 2011

CORRECTORS FOR A HOMOGENIZATION PROBLEM IN AN OSCILLATORY THIN DOMAINS

MARCONE C. PEREIRA* & RICARDO P. SILVA †

In this work we analyze convergence properties of solutions of the family of problems

$$\begin{cases} -\Delta w^\epsilon + w^\epsilon = f^\epsilon & \text{in } R^\epsilon \\ \dfrac{\partial w^\epsilon}{\partial N^\epsilon} = 0 & \text{on } \partial R^\epsilon \end{cases} \tag{0.1}$$

where $R^\epsilon = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 \in (0,1),\ 0 < x_2 < \epsilon\, g(x_1/\epsilon)\}$ is a thin domain with part of its boundary highly oscillating driven by a L-periodic function $g \in C^1(\mathbb{R})$, N^ϵ is the unit outward normal vector field to ∂R^ϵ and $f^\epsilon \in L^2(R^\epsilon)$.

It is important to notice that the amplitude and period of the oscillations are of the same order ϵ, which also coincides with the order of thickness of the thin domain. This scaling makes the problem very resonant and the determination of the limiting problem not straight forward.

Results recently obtained in [2] show that w^ϵ converges to a "one-dimensional" limit $w_0 \in H^1(0,1)$ in a weak topology of H^1, whereas in [1] the author shows that strong convergence is actually false. For this reason, we use an appropriate *corrector approach* developed in [3] to obtain a kind of strong convergence. This is made introducing some *correctors*, $\kappa^\epsilon \in H^1(R^\epsilon)$, $\kappa^\epsilon = \mathrm{o}(\epsilon)$ in $L^2(R^\epsilon)$ in order to

$$\epsilon^{-1/2}\|w^\epsilon - w_0 - \kappa^\epsilon\|_{H^1(R^\epsilon)} \to 0.$$

References

[1] J. M. ARRIETA, *Spectral properties of Schrodinger operators under perturbations of the domain.* Ph.D. Thesis, Georgia Institute of Technoly (1991).

[2] J. M. ARRIETA AND A. N. CARVALHO AND M. C. PEREIRA AND R. P. SILVA, *Nonlinear parabolic problems in thin domains with a highly oscillatory boundary*, Nonlinear Analysis (74) (2011) 5111-5132 .

[3] A. BENSOUSSAN AND J. L. LIONS AND G. PAPANICOLAOU, *Asymptotic Analysis for Periodic Structures*, North-Holland (1978).

[3] M.C. PEREIRA AND R.P. SILVA, *Error estimates and homogenization for a Neumann problem in highly oscillating thin domains*, submitted.

*Escola de Artes, Ciências e Humanidades, Universidade de São Paulo, São Paulo SP, Brazil, e-mail: marcone@usp.br
†Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, Rio Claro SP, Brazil, e-mail: rpsilva@rc.unesp.br