EBED - Escola Brasileira de Equações Diferenciais
UFPB - Universidade Federal do Paraíba
Edição N$^{\underline{0}}$ 4 Agosto 2011

SINGULAR SOLUTIONS OF FULLY NONLINEAR EQUATIONS IN CONES AND APPLICATIONS TO ELLIPTIC SYSTEMS

BOYAN SIRAKOV*

In this talk I briefly describe the two recent works [1], [2]. In the first one we proved that any fully nonlinear, positively 1-homogeneous elliptic operator (for instance, a Pucci extremal operator) has a positively homogeneous and a negatively homogeneous "fundamental" solutions in each cone of $\mathbb{R}^N$. In [2] we developed a new and optimal method for proving nonexistence of solutions of elliptic inequalities of the type $-Qu \geq f(x,u) \geq 0$ in unbounded domains, in the presence of a fundamental subsolution of Q. These results can be combined to give a (partial) answer to a long standing open problem in the theory of elliptic systems, namely existence of solutions of superlinear systems modelled on

$$\begin{cases} -L^{(1)}u &= v^p & \text{in} & \Omega \\ -L^{(2)}v &= u^q & \text{in} & \Omega \\ u,v &> 0 & \text{in} & \Omega \\ u,v &= 0 & \text{on} & \partial\Omega, \end{cases} \tag{0.1}$$

where Ω is a smooth bounded domain and $L^{(k)}$ are uniformly elliptic partial differential operators with bounded coefficients

$$L^{(k)}u := \sum_{i,j=1}^{N} a_{ij}^{(k)}(x)\partial_{ij}u + \sum_{i=1}^{N} b_i^{(k)}(x)\partial_i u + c^{(k)}(x)u, \quad k = 1,2.$$

Existence results were previously available only in the case when the two operators $L^{(1)}, L^{(2)}$ have *identical principal parts* (that is, the matrices $(a_{ij}^{(1)})$, $(a_{ij}^{(2)})$ are proportional). Thanks to the results mentioned above, we show there exist ranges of $p, q > 0$ in which (0.1) has positive solutions for arbitrary $L^{(1)}, L^{(2)}$.

References

[1] S.N. AMSTRONG, B.SIRAKOV, AND C.K. SMART, Singular solutions of fully nonlinear elliptic equations and applications, http://arxiv.org/abs/1104.5338
[2] S.N. AMSTRONG, B. SIRAKOV, Nonexistence of positive supersolutions of elliptic equations via the maximum principle, *to appear in Comm. Part. Diff. Eq.* http://arxiv.org/abs/1005.4885

*Université Paris Ouest, France, e-mail: sirakov@ehess.fr