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GLOBAL SOLVABILITY OF A SYSTEM OF LPDE'S ON THE TORUS $\mathbb{T}^{n+1}$

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In this work we study the global solvability of a system of n smooth vector fields on the torus $\mathbb{T}^{n+1} \simeq (\mathbb{R}/2\pi\mathbb{Z})^{n+1}$ given by

$$L_j = \frac{\partial}{\partial t_j} + (a_j(t) + ib_j(t_j))\frac{\partial}{\partial x}, \;\; j = 1, \dots, n,$$

where $a_j, b_j \in \mathcal{C}^\infty(\mathbb{T}^n; \mathbb{R})$ and $(t, x) = (t_1, \dots, t_n, x)$ are the coordinates in the torus $\mathbb{T}^{n+1}$. We assume that the 1-form $c = a + ib \in \bigwedge^1 \mathcal{C}^\infty(\mathbb{T}^n)$ is closed, where $a(t) = \sum_{j=1}^n a_j(t)dt_j$ and $b(t) = \sum_{j=1}^n b_j(t_j)dt_j$ are real 1-forms. Therefore, the system is involutive (see [7]). We study the global solvability in the following sense: Given functions $f_1, \dots, f_n$ in $\mathcal{C}^\infty(\mathbb{T}^{n+1})$ satisfying certain compatibility conditions, we seek necessary and/or sufficient conditions for the system of the differential equations

$$L_j u = f_j, \quad j = 1, \dots, n,$$

to have a solution u in $\mathcal{D}'(\mathbb{T}^{n+1})$.

We consider the case where each function b_j is not identically zero. If b is non-exact then the global solvability is completely determined by the existence of a function b_j that does not change sign. Also, we prove that this condition is equivalent to the property of all the sublevel and superlevel sets of the global primitive of $\Pi^* b$ being connected in the minimal covering space $\Pi : \mathcal{T} \to \mathrm{T}^n$ on which $\Pi^* b$ is exact. If b is exact then the system is globally solvable if and only if the real part a is integral and all the sublevel and superlevel sets of the global primitive of b are connected in $\mathbb{T}^n$.

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