EBED - Escola Brasileira de Equações Diferenciais
UFPB - Universidade Federal do Paraíba
Edição N$^{\underline{0}}$ 4 Agosto 2011

On the least energy level of potential type functionals on maps spaces

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In this work we study in depth the least energy level of the functional

$$\Phi_G(U) = \int_\Omega |\nabla U|^2 \, dx - \int_\Omega G(U) \, dx$$

constrained to the Nehari manifold $E_F = \left\{ U \in W_0^{1,2}(\Omega, \mathbb{R}^k) : \int_\Omega F(U) \, dx = 1 \right\}$, where $\Omega \subset \mathbb{R}^n$ is a bounded open, $n \geq 3$, $G : \mathbb{R}^k \to \mathbb{R}$ is a continuous function being 2-homogeneous and $F : \mathbb{R}^k \to \mathbb{R}$ is a positive continuous function being 2^*-homogeneous with $2^* = \frac{2n}{n-2}$. Let

$$C_{F,G} := \inf_{U \in E_F} \Phi_G(U) \, .$$

A simple argument immediately yields $C_{F,G} \leq M_F^{2/2^*}/K(n,2)^2$, where $K(n,2)$ is the best constant for the embedding $\mathcal{D}^{1,2}(\mathbb{R}^n) \hookrightarrow L^{2^*}(\mathbb{R}^n)$. Consider the least minimal energy set $X_{F,G} = \{U \in E_F : \Phi_G(U) = C_{F,G}\}$. In this broad context, without assuming smoothness, we establish compactness and concentration results and De Giorgi-Nash-Moser estimates for sequences in $X_{F,G}$.

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