EBED - Escola Brasileira de Equações Diferenciais
UFPB - Universidade Federal do Paraíba
Edição N$^{\underline{0}}$ 4 Agosto 2011

GEOMETRIC APPROACH TO NONVARIATIONAL SINGULAR ELLIPTIC EQUATIONS

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We study geometric and sharp smoothness properties of fully nonlinear elliptic equations with singular potentials:

$$F(D^2u, Du, X) \approx \gamma |u|^{\gamma-2}u, \quad 0 < \gamma < 1. \tag{0.1}$$

No sign constraint is assumed, thus the above PDE should be regarded as a two-phase free boundary problem. The variational theory is fairly well understood, nowadays. It appears as the Euler-Lagrange equation in the minimization of non-differentiable functions:

$$\int (\nabla u)^2 + u^\gamma dX \longrightarrow \min.$$

It has been shown (see [1], [2], [5]) that minimizers of the above functional are locally $C^{1,\frac{\gamma}{2-\gamma}}$ and such a regularity is optimal.

In this work, under natural conditions on F, we show existence of minimal solutions to Equation 0.1. We also prove that minimal solutions are locally $C^{1,\frac{\gamma}{2-\gamma}}$, provided F has *a priori* $C^{1,1}$ estimates. In addition we show that minimal solutions grow at the optimal possible rate away from the free boundary $\mathcal{F} := \partial\{u > 0\} \cap \partial\{u < 0\}$, that is: if $X \in \{\pm u > 0\}$, then

$$u^{\pm}(X) \geq \mu \mathrm{dist}(X, \mathcal{F})^{\frac{2}{2-\gamma}},$$

for some universal constant $\mu > 0$. This estimate gives an appropriate nondegeneracy property of u along the free boundary, which allows the weak geometry analysis of $\mathcal{F}$, in particular Hausdorff estimates of the free boundary.

Inspired by a varying singularity technic [3], we also provide a through analysis of asymptotic free boundary problems obtained as $\gamma \nearrow 1$ and $\gamma \searrow 0$. The former gives a fully nonlinear obstacle type problem and the latter provides a path to understanding two-phase fully nonlinear free boundary problems of the Alt-Caffarelli-Friedman cavity type. The one-phase case has been recently studied in [4].

References

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